VERIFICATION OF A MONTE CARLO SIMULATION METHOD TO FIND LOWER CONFIDENCE LIMITS FOR THE AVAILABILITY AND RELIABILITY OF MAINTAINED SYSTEMS

THESIS

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Preface

With the increasing complexity and astronomical costs associated with the design and production of space systems, availability and reliability are among the most important factors to consider when building space systems. Also, as repair of space systems becomes more feasible with the advent of the shuttle and other upcoming space projects, maintainability is becoming another important factor in space system design. But none of these factors can be applied without a thorough understanding of the theory behind availability, reliability, and maintainability.

The purpose of this study is to verify the hypothesis that Monte Carlo simulation is the best method of finding lower confidence bounds for the availability and reliability of maintained systems in different configurations. Also, this study verifies that once a lower confidence bound is found, it is the same over the lifetime of a maintained system.

I would like to thank my thesis advisor, Professor Albert H. Moore, for his most valuable and very expert advice and guidance during this study. I would also like to thank Lt Col Joseph W. Coleman, my reader, for his help and

guidance during this study. Finally, I wish to thank my husband, Nick, for his encouragement and moral support during this study and throughout this master's program.

Karen S. Harland

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Abstract

This ~~research~~ thesis determined the feasibility and efficiency of a Monte Carlo method of simulating the lower confidence limits for the availabilities and reliabilities of maintained systems. The steady-state availabilities of single-unit systems and the time-constrained availabilities and reliabilities of two-unit parallel systems were simulated.

First, a baseline of "true" exponentially-distributed Mean Time Between Failures (MTBFs) and Mean Time To Repairs (MTTRs) were simulated using the chi-square distribution. Then other MTBFs and MTTRs were simulated to represent sampling of other systems. The availabilities and reliabilities were found using these simulated MTBFs and MTTRs. Next, simulated availabilities and reliabilities were ordered, and lower confidence limits were found. These lower confidence limit point estimates were compared against the systems' exact availabilities and reliabilities. Lastly, the success of this Monte Carlo method is determined by how well the simulated lower confidence limit availability and reliability point estimates cover the exact availabilities and reliabilities.

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I. Introduction

The reliability and maintainability disciplines became firmly established in the 1950's due to the United States' increasing emphasis on its military and space programs. The need for engineers to develop more efficient and longer-lasting equipment for these programs was due to several factors:

a) high equipment failure rates
b) inflation which increased the cost of buying and maintaining equipment
c) increased equipment complexity
d) the desire to develop a methodical approach to minimize causes of failure (Fullerton, 1969:1).

Then and now, engineers are interested in three general areas when designing systems - reliability, maintainability, and availability. Reliability is "the probability that, when operating under stated environmental conditions, the system will perform its function adequately for a specified interval of time" (Kapur,Lamberson, 1977:1). Maintainability is "the probability that a failed system can be made operable in a

specified interval of downtime" (Kapur,Lamberson, 1977:225). Thus, a maintained system is one that can be repaired. The measure that includes both reliability and maintainability is availability. Availability is "the probability that a system is operating satisfactorily at any point in time and considers only operating time and downtime, thus excluding idle time" (Kapur,Lamberson, 1977:225). An engineer can be sure his design is sound if he can measure the reliability and availability of the system, and those measures fall into his specified confidence interval. A confidence interval is

> a range of values which is believed, with a preassigned degree of confidence, to include the particular value of some parameter or characteristic being estimated. The degree of confidence is related to the probability of obtaining by random samples ranges which are correct [James,Beckenbach, 1968:65].

Many statisticians have done previous work in developing and evaluating techniques to obtain the confidence limits of system reliabilities and availabilities modeled by different underlying distributions. One of the first works presented came from Mary Thompson. Thompson developed analytical techniques to determine confidence limits for the availability of exponentially distributed repairable systems in 1966 (Thompson, 1966). In the following year, Louis Levy and Albert Moore introduced a Monte Carlo simulation technique to estimate reliability confidence limits from component test data with normal, gamma, or Weibull

probability distributions (Levy,Moore, 1967). H. L. Gray and W. R. Schucany extended Thompson's techniques to find availability confidence limits for systems with lognormally distributed repair times in 1969 (Gray,Schucany, 1969). However, work has yet to be done to obtain confidence limits for the availability and reliability of maintained systems using Monte Carlo simulation with simulated system data.

Statement of the Problem

Markov models and exact analytical techniques have been used to produce confidence limits for the reliability and availability of maintained systems. However, as system size and complexity grows, these models and techniques become very hard to apply. A quick and efficient Monte Carlo simulation method is needed to find confidence limits for the reliability and availability of maintained systems, and an assessment of the accuracy of the method is needed. According to current literature, a Monte Carlo simulation can identify independent or dependent system failures occurring in either simple or complex systems (Almassy, 1979:366). Monte Carlo simulation can also model systems with general failure and repair distributions. (In this case, the failure and repair distributions are assumed to be exponential because the exponential distribution accurately models the failure of most complex electronic systems and provides a fair model for the repair of electronic systems (Thompson, 1966:36).)

Therefore, Monte Carlo simulation will provide an easy and accurate method of obtaining reliability and availability confidence limits for maintained systems.

Objectives of the Research

The overall objective of this research is to develop an efficient Monte Carlo simulation to find confidence limits for the reliability and availability of maintained systems. The subobjectives for this research are as follows:

a) to develop a Monte Carlo simulation method to find availability confidence limits for small sample, single system configurations

b) to find the availability for a single system configuration system configuration using an exact analytical technique

c) to evaluate the accuracy of the Monte Carlo simulation method by comparing the Monte Carlo confidence limits to the exact availability for a single system configuration

d) to expand the Monte Carlo simulation to get both reliability and availability confidence limits for small sample, parallel system configurations with time and, if time permits, more complex systems

e) to present the actual percentage coverage of the confidence intervals of the complex systems as another way of determining the accuracy of this

method of obtaining confidence limits.

Methodology

The overall approach for this research is to use a Monte Carlo simulation to estimate the availabilities and reliabilities of maintained systems and determine the lower confidence limits for both. This Monte Carlo simulation will generate sample Mean-Time-To-Repair (MTTR) and Mean-Time-Between-Failure (MTBF) values from the chi-square distribution. The MTTRs and MTBFs are exponentially distributed and are from small samples (ten to fifty system configurations). The samples are Type II censored samples which are a set of n items that are tested until a specific number of failures have occurred (Moore, 1965:460). Input sample MTTR (ϕ) and MTBF (θ) values will form the "real world" baseline data against which the rest of the values will be compared.

The simulation will initially consist of two parts. First, only the availability of a single system configuration will be estimated using the generated MTTRs and MTBFs. The estimated availability is found from the following equation (Kapur,Lamberson, 1977:228)

$$A_{ss} = MTBF/(MTBF + MTTR) \qquad (1)$$

The reliability of a single system configuration will not be estimated because if a system is not available, it has

failed, and at that point, it has reached its reliability limit. However, for the second part of the simulation, both the availability and reliability of two-unit parallel systems with repair will be estimated, again by using the generated MTTRs and MTBFs. The availability and reliability equations for parallel systems are as follows (Shooman, 1966:341-346):

$$A(t) = (1-(\lambda\lambda'/r_3r_4)) - \lambda\lambda'/r_3r_4 \quad (e^{r_3t}/r_3 - e^{r_4t}/r_4) \qquad (2)$$

where

$\lambda = 1/\theta$

$\mu = 1/\phi$

$\lambda' = 2\lambda$ for an ordinary system

$\lambda' = \lambda$ for a standby system

$\mu' = \mu$ for one repairman

$\mu' = k_1\mu$ for more than one repairman $(k_1 > 1)$

$\mu'' = \mu$ for one repairman

$\mu'' = 2\mu$ for two repairmen

$$\mu'' = k_2\mu \quad \text{for more than two repairmen}$$

$$(k_2 > 2)$$

$$r_3, r_4 = [-(\lambda + \lambda' + \mu' + \mu'') +/- $$
$$(((\lambda + \lambda' + \mu' + \mu'')**2)$$
$$- (4(\lambda\lambda' + \lambda'\mu'' + \mu'\mu''))^{1/2}]/2$$

$$R(t) = (\lambda + \mu' + r_1/r_1 - r_2)e^{r_1 t} \tag{3}$$
$$- (\lambda + \mu' + r_2/r_1 - r_2)e^{r_2 t}$$
$$+ (\lambda'/r_1 - r_2)e^{r_1 t}$$
$$- (\lambda'/r_1 - r_2)e^{r_2 t}$$

where

$$r_1, r_2 = [-(\lambda + \lambda' + \mu') +/- ((\lambda + \lambda' + \mu')**2$$
$$- 4\lambda\lambda')^{1/2}]/2$$

$\lambda' = 2\lambda$ for an ordinary system

$\lambda' = \lambda$ for a standby system

$\mu' = \mu$ for one repairman

$\mu' = k\mu$ for more than one repairman $(k > 1)$

Reliabilities can be found for the two-unit system configuration because one unit in a parallel configuration may fail, but its parallel unit may still be available, so the configuration still works. Thus, the parallel system configuration has a reliability that differs from the availability. (The repairable, parallel, single repairman configuration will be considered in this simulation; others will be considered as time permits.)

After both parts of the simulation have been run, the lower confidence limits are found and, by simulating a large number of trials, a higher level of accuracy in estimation can be obtained. This Monte Carlo simulation method can be assessed and verified by how times the confidence interval contains the true system availabilities and reliabilities.

Sequence of Presentation

The next chapter summarizes the literature available on this subject from the 1960s to the present. Chapter 3 presents the Monte Carlo simulation models designs and the equations and variables used in the model. Chapter 4 discusses the simulation results in answer to the thesis subobjectives and conclusions. Lastly, Chapter 5 presents a summary of the research effort and recommendations for future studies.

II. Literature Review

As previously stated in Chapter I, statisticians and theorists have been developing various techniques to find availabilities and reliabilities of many different system configurations with many different underlying failure or repair distributions. This chapter will review some of that work, divided into these different areas as much as possible: availability, reliability, Monte Carlo simulation of availability or reliability, and classic books.

Works on Availability

There have been several works published focusing strictly on availability since the mid-1960s. In the year following Thompson's work on analytically determined availability confidence limits, John Buzacott showed how to reduce reliability block diagrams of repairable series-parallel systems to find the MTBF and availability of the system components (Buzacott, 1967). Also in 1967, H. L. Gray and T. O. Lewis presented an analytical method to find exact system availability confidence limits with exponentially distributed MTBFs and lognormally distributed MTTRs (Gray,Lewis, 1967). In 1969, Kenneth Grace gave a comparison between a Markov model and some approximation techniques used to find the steady-state availability of repairable systems with limited component spares (Grace,

1969). In 1970, Buzacott did some more research on repairable systems, this time using network diagrams to find system availability and failure frequencies (Buzacott, 1970b). In 1975, David Chow presented a mathematical model to find the availabilities of a redundant, repairable system with a standby. The system had to have q components available to work, and the underlying failure rates were constant with generally distributed MTTRs (Chow, 1975). Several years later and building on the works of Thompson, Gray and Lewis, and Gray and Schucany, Mohamed Hasaballa, Albert Moore, and Joseph Cain presented an exact analytical method of finding lower confidence limits on steady-state asymptotic availabilities of systems with exponentially distributed MTBFs, and exponentially and lognormally distributed MTTRs (Hasaballa and others, 1983). In 1983, another work concerning repairable systems was published by Richard Kenyon and Richard Newell. They gave an exact analytical solution and a Fortran program designed to find the steady-state availability of a system needing k-out-of-n components to function, but allowing only one repair for the system lifetime (Kenyon,Newell, 1983). During the same year, Espen Funnemark and Bent Natvig broke new ground with their presentation on how to find upper and lower confidence limits for the availability of multi-state component systems (Funnemark,Natvig, 1983). Lastly, in 1984, Ignacio Mendez introduced a method of finding the steady-state availability

for n unlike parallel or n unlike series components with exponentially distributed failure and repair rates (Mendez, 1984). All the above works have bearing on this thesis because they've considered repairable or complex systems, exponentially distributed failure or repair rates, confidence limits, or steady-state availability. The next section presents reliability works which have considered the same factors.

Works on Reliability

The list of theorists working on reliability prediction techniques is quite extensive, and this section gives only the highlights. Starting in the early 1960s, Thomas Burnett and Beverly Wales presented an analytical method and a simple Monte Carlo method to determine system reliability confidence limits from component test results (Burnett,Wales, 1961). In 1963, Oscar Bernhoff researched a method to analytically determine system reliability confidence limits by combining the components' reliability estimates to get the overall failure probability density function. He also determined that a Monte Carlo simulation of the overall system distribution was much easier to use when component estimates were from two or more dissimilar distributions (Bernhoff, 1963). Also in 1963, Malcolm McGregor worked out a method of determining the reliabilities of repairable systems with n identical parallel components having exponentially

distributed MTBFs and MTTRs (McGregor, 1963). In 1965, Albert Madansky revealed how to combine component reliability estimates gathered by separate tests to determine the overall complex system reliability (Madansky, 1965). Then, in 1966, L. Htun used McGregor's findings in his work about using transition diagrams describing different states of repairable or nonrepairable systems to determine reliability (Htun, 1966). In 1970, Buzacott presented yet another study, this time on special Markov techniques for determining the reliability (and availability) for a large number of states in repairable systems (Buzacott, 1970a). (Earlier, in 1966, Kenneth Blakney and Frederick Dietrich presented a thesis covering Markov reliability processes known to that time; R. Fullerton updated that list in 1969 [Blakney,Dietrich, 1966; Fullerton, 1969].) Then, in 1972, building on Madansky's work, Robert Easterling presented a technique for determining the maximum likelihood estimate of system reliability from component test results and substituting that estimate into an incomplete beta function to determine reliability confidence limits (Easterling, 1972). Next, Nancy Mann and Frank Grubbs presented several methods of approximating reliability confidence limits for series or parallel systems using component test results gathered from Type I or Type II censored sampling; they based their work on studies done previously by Madansky and Easterling (Mann,Grubbs, 1974). Later, in 1977 and 1978, Edward Bilikam and Albert Moore

published a extended study on estimating reliability confidence limits from multiple independent grouped censored samples with failure times known or unknown (Bilikam,Moore, 1977 and 1978). Looking again at the technique of using Markov processes to determine system reliability, Joseph Foster and Alberto Garcia-Diaz formulated generalized Markov models for three classes of reliability: systems with catastrophic failure, systems that must be down to be repaired, and systems that can be repaired while functioning (Foster,Garcia-Diaz, 1982). Again, in the area of multi-state systems, David Butler presented mathematical computations for finding complex multi-state system reliability bounds in 1982 (Butler, 1982). In the same year, I. Gertsbakh also presented mathematical formulas to determine upper reliability confidence limits for parallel, series-parallel, and k-out-of-n systems, all with exponential component distributions, and Type I or Type II censored sampling (Gertsbakh, 1982). More mathematical formulas were presented by Tetsuo Miyamura for combining exponentially distributed component and system estimates to determine component reliability (Miyamura, 1982). Lastly, one of the more recent works was presented by Tze Li in 1984 when he demonstrated the use of an empirical Bayes estimator for determining system reliability using a large sample size of systems with exponentially distributed failure rates (Li, 1984).

Monte Carlo Computer Simulation Works

The projects presented in this section represent work done in the areas of availability or reliability specifically using Monte Carlo simulation techniques. Beginning in 1960, Donald Orkland published a pioneer simulation study on finding lower confidence limits for system reliability using only sample component failure data (Orkland, 1960). Two years later, W. Connor and W. Wells expanded Orkland's work with a study of their own which found system reliability confidence intervals for serially arranged components with binomially distributed failure rates (Connor,Wells, 1962). Next, in 1964, Louis Levy devised a Monte Carlo method to find system reliability confidence limits using component test data with exponential, Weibull, gamma, normal, or lognormal failure rates. He based his method on previous works by Burnett and Wales, and Bernhoff (Levy, 1964). The following year, Albert Moore presented an extension of the Monte Carlo technique used by Levy to find confidence limits when the distribution or joint distribution of the estimators were unknown and the data came from Type I or Type II censored sampling (Moore, 1965). Then, in 1967, Levy and Moore presented a joint paper on a more efficient Monte Carlo simulation technique to estimate reliability confidence limits from component test data having normal, gamma, or Weibull failure distributions (Levy,Moore, 1967). In 1968, Leonard Doyon and Martha Berssenbrugge presented a

computational method which represented system states as differential equations and iteratively calculated reliability and availability estimates; they based their work partly on Htun's work (Doyon,Berssenbrugge, 1968). During that same year, Donald Gilmore presented a study that used Monte Carlo methods to simulate failures of a component reliability block diagram to estimate the overall complex system reliability (Gilmore, 1968). In 1972, Robert Lannon devised a Monte Carlo technique to determine the reliability of a complex system with dissimilar components having Weibull failure distributions. Previous studies by Bernhoff, and Levy and Moore provided a basis for his work (Lannon, 1972). Next, over the period of two years, Satish Kamat, Michael Riley, and William Franzmeier jointly published studies on using Monte Carlo simulation to find complex system reliability by inputting reliability flow graphs and minimal tie-sets (Kamat,Riley, 1975; Kamat,Franzmeier, 1976). Next, Hiromitsu Kumamoto, Kazuo Tanaka, and Koichi Inoue published a study based on previous work done by Levy and Moore, and Kamat, Riley, and Franzmeier. They developed a Monte Carlo simulation to find system reliability using reliability block diagrams or fault tree analysis with a controlled variate or importance sampling technique (Kumamoto and others, 1977). In 1980, Malcolm Easton and C. Wong used a sequential destruction method to estimate reliability for systems of 100 or more components and with dependent or independent

component failures; their work is based on the work of Kumamoto, Tanaka, and Inoue (Easton,Wong, 1980). In turn and in the same year, Kumamoto, Tanaka, Inoue, and Ernest Henley presented a study based on Easton and Wong's work which found the unavailability of large, complex systems by using dagger sampling which decreased the number of trials and computation time of the Monte Carlo simulation (Kumamoto and others, 1980a). They also published another study in late 1980 which gave a technique to estimate the unreliability of large, repairable systems using a state transition or indirect Monte Carlo method (Kumamoto and others, 1980b). Two years later, another foursome presented a study on three different Monte Carlo techniques used to find system reliability confidence limits; Kathleen Depuy, Jon Hobbs, Albert Moore, and J. Johnston developed and analyzed the accuracy of univariate asymptotic, bivariate asymptotic, and 'modified double Monte Carlo' techniques. They based their work on studies done by Orkland, and Levy and Moore (Depuy and others, 1982). In 1983, Roy Rice and Albert Moore published a paper describing a Monte Carlo simulation using pass-fail data, and they explained how to treat no failure cases. Their work was based, in part, on Easterling's, Orkland's, and Levy and Moore's previous works (Rice,Moore, 1983). Lastly, one of the latest works was presented in April, 1985 by Kadaba Gopalakrishnan who developed a Monte Carlo model to find general system reliability, maintainability, and availability

with interruption factors and delays in repair taken into account. His model used findings previously given by Gilmore, Kamat, Riley and Franzmeier, and Buzacott (Gopalakrishnan, 1985).

Classical Books

Although not previously mentioned, the basis for many of the articles on availability, reliability, and Monte Carlo simulation summarized above came from two books. The first book, Mathematical Theory of Reliability, was written by Richard Barlow and Frank Proschan in 1965 and presented basic mathematical theories for reliability (Barlow, Proschan, 1965). The second book, Probabilistic Reliability: An Engineering Approach, was written by M. L. Shooman in 1968 which gave an engineering perspective to reliability (i.e., how to apply reliability in engineering) (Shooman, 1968). Both books are classics because they helped lay the basic groundwork from which other studies were generated.

Conclusion

Many studies have been done in the areas of system and component availability and reliability, but many more are yet to be done as today's equipment and systems grow even more complex.

III. The Monte Carlo Simulation Model

This chapter describes the design and computations used in the Monte Carlo simulation model. The simulation model was used to find the availabilities of single-unit systems, and the availabilities and reliabilities of two-unit ordinary parallel systems.

Single-Unit System

Computations. The simplest system to analyze is a single-unit system. This Monte Carlo model simulates the availability (A(t) where t is time) of single-unit systems having an exponential, or constant, failure distributions. Now, if the single-unit system has no capability to be repaired, the system availability will equal the system reliability. But, having a repair capability increases the system availability - a desirable system characteristic. The model computation calculates the availability for a repairable system which will be larger than the system reliability. A Markov graph models the states of single-unit system availabilities in Figure 3.1 (Shooman, 1968:338).

The symbols in Figure 3.1 are

x_1 = the element (or system) in a state

λ = 1/Mean Time Between Failure (MTBF)

μ = 1/Mean Time To Repair (MTTR)

s_0 = the operating state of the system

s_1 = the failure state of the system.

Δt = change in time

(If the system was not repairable, s_1 would be an "absorbing" state, and no return to s_0 would be possible .)

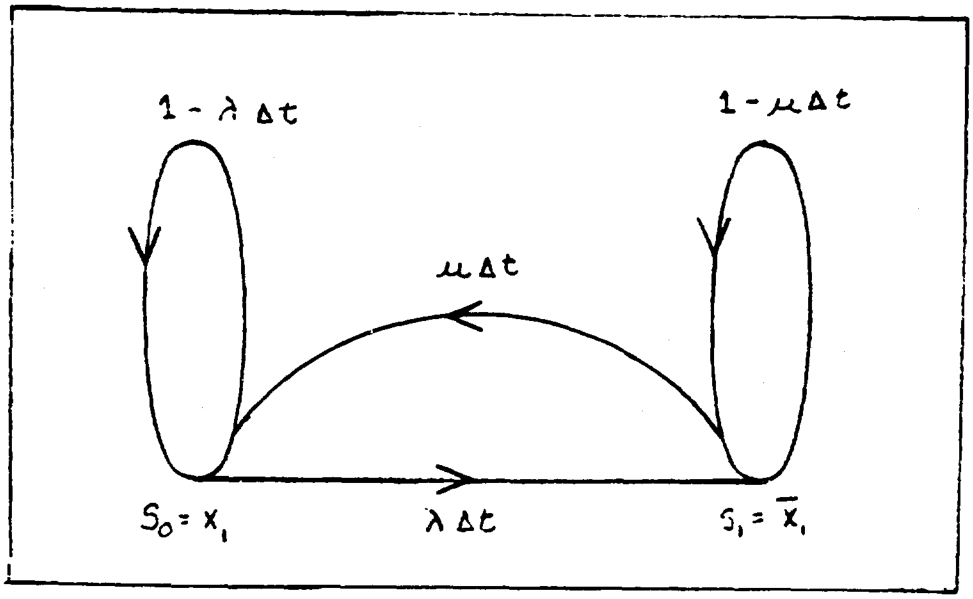

Figure 3.1 Markov Graph for Single-Unit System Availability (Shooman, 1968:338)

The Markov graph state probability equations used in this simulation model are as follows:

$$Ps_0(t) = \mu/(\lambda+\mu) + (\lambda/(\lambda+\mu)(\exp(-(\lambda+\mu)(t)))) \quad (4)$$

$$Ps_1(t) = \lambda/(\lambda+\mu) - (\lambda/(\lambda+\mu)(\exp(-(\lambda+\mu)(t)))) \quad (5)$$

So, by definition, the availability (or probability of the system being operational) is $Ps_0(t)$.

$$A(t) = Ps_0(t) \quad (6)$$

Now, as t (or time) gets larger, the availability function approaches approaches a certain value - a "steady-state" value. For the single-unit system, the steady-state availability ($A_{ss}(t)$) is

$$A_{ss}(t) = \lim A(t) = \mu/(\lambda+\mu) \quad (7)$$

Equation 7 is the equation used in the Monte Carlo model to compute the steady-state availabilities of sample single-unit systems (Shooman, 1968:338).

Model. The simulation model used the following baseline parameters:

Mean Time Between Failure ($1/\lambda$) = 100 hours

Mean Time To Repair ($1/\mu$) = 20 hours

First, new estimates of MTBFs (θ) and MTTRs (ϕ) were generated, thus simulating MTBFs and MTTRs from "true" single-unit systems. These new MTBFs and MTTRs were then used to simulate new sample MTBFs and MTTRs. Both samples were drawn using the chi-square distribution as follows:

$$\theta = (X^2_{2r}\theta)/2r \qquad (8)$$

where

r = number of failures for the Type II censored sample

X^2_{2r} = random number drawn from the chi-square distribution with 2r degrees of freedom

and MTTRs were generated using the same equation with substituted for θ.

Then, each sample θ and ϕ were converted to λ and μ respectively ($\lambda = 1/\theta$ and $\mu = 1/\phi$). Finally, the simulated steady-state availability point estimates were

found using

$$A_{ss}(t) = \mu / (\lambda + \mu) \quad (9)$$

In summary, simulated steady-state availabilities were found for Type II censored sample sizes of 10, 20, 30, and 50. Point estimates were found for the lower confidence limits of 0.98, 0.95, 0.90, 0.85, and 0.80. The model ran for 500 repetitions with 500 trials per repetition.

Two-Unit Parallel System

This Monte Carlo model simulates the availabilities (A(t)) and reliabilities (R(t)) of samples of two-unit ordinary identical parallel systems. The systems have both exponential, or constant, failure and repair rates. The computations and model for both availability and reliability are discussed below.

Availability Computation. Due to the nature of it's configuration, the parallel system can have a repair capability that increases it's overall availability. If one unit in the system fails, the system can keep functioning with the operational unit while the failed unit is being repaired. The different availability states that a parallel system can be in are shown in the following Markov graph in

Figure 3.2 (Shooman, 1968:344).

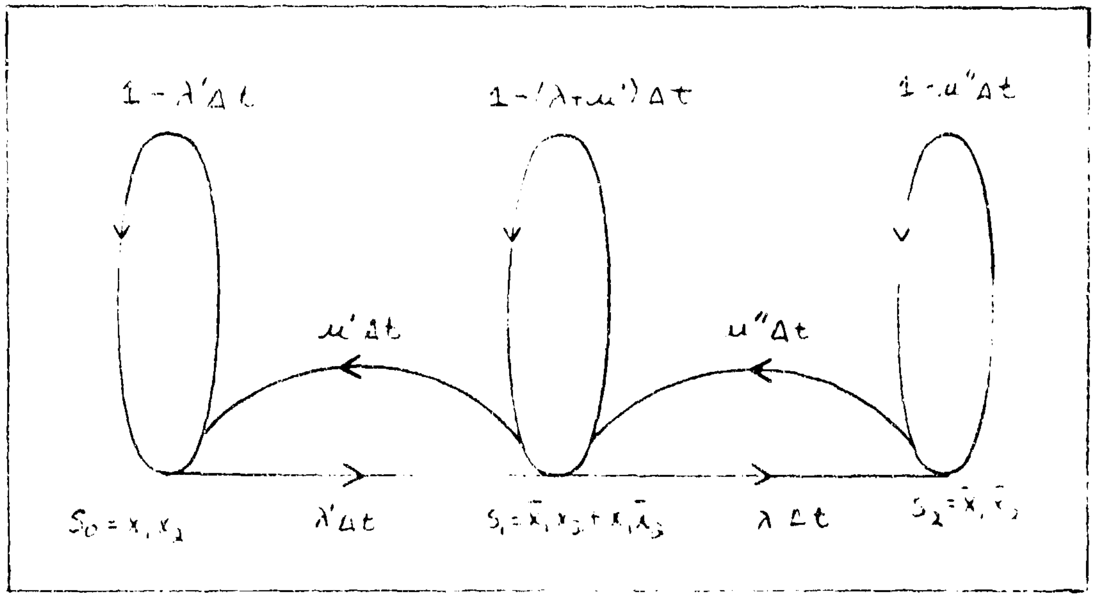

Figure 3.2 Markov Graph for Two-Unit Parallel System Availability with Repair (Shooman, 1968:344)

The symbols in Figure 3.2 are defined as follows:

s_0 = both units operational

s_1 = one unit failed, one unit operational

s_2 = both units failed

x_1, x_2 = unit designators

λ = 1/Mean Time Between Failure or $1/\theta$

μ = 1/Mean Time To Repair or $1/\phi$

$\lambda' = 2\lambda$ for an ordinary system

$\lambda' = \lambda$ for a standby system

$\mu' = \mu$ for one repairman

$\mu' = k_1\mu$ for more than one repairman ($k_1 > 1$)

$\mu'' = \mu$ for one repairman

$\mu'' = 2\mu$ for two repairmen

$\mu'' = k_2\mu$ for more than two repairmen ($k_2 > 2$)

Δt = change in time

Note that the Markov graph accounts not only for the failure of zero, one, or both units, but that it also accounts for a differing number of repairmen working on the system. This is an important point to consider for any repairable system. If too many repairmen are working on a system, a point of diminishing returns is reached when the repairmen begin interfering with each other's work. By varying the number of repairmen in the model, their effect on system availability can be simulated, and an optimum number of repairmen can be found.

Finally, the availability equation used in this model is derived from Figure 3.2 (Shooman, 1968:345-346). The equation is

$$A(t) = (1 - \lambda\lambda'/r_3r_4) - \lambda\lambda'/r_3 - r_4(e^{r_3t}/r_3 - e^{r_4t}/r_4) \qquad (10)$$

where

$$r_3, r_4 = [-(\lambda + \lambda' + \mu' + \mu'')$$
$$\quad +/- \quad ((\lambda + \lambda' + \mu' + \mu'')^2$$
$$\quad - 4(\lambda\lambda' + \lambda'\mu'' + \mu'\mu''))**(1/2)]1/2$$

and other symbols are as defined for Figure 3.2

<u>Reliability Computation</u>. Like the availability of a parallel system, the reliability is better when the parallel system has a repair capability. The Markov graph in Figure 3.3 shows the different reliability states that a repairable parallel system can be in and transition to (Shooman, 1968:341).

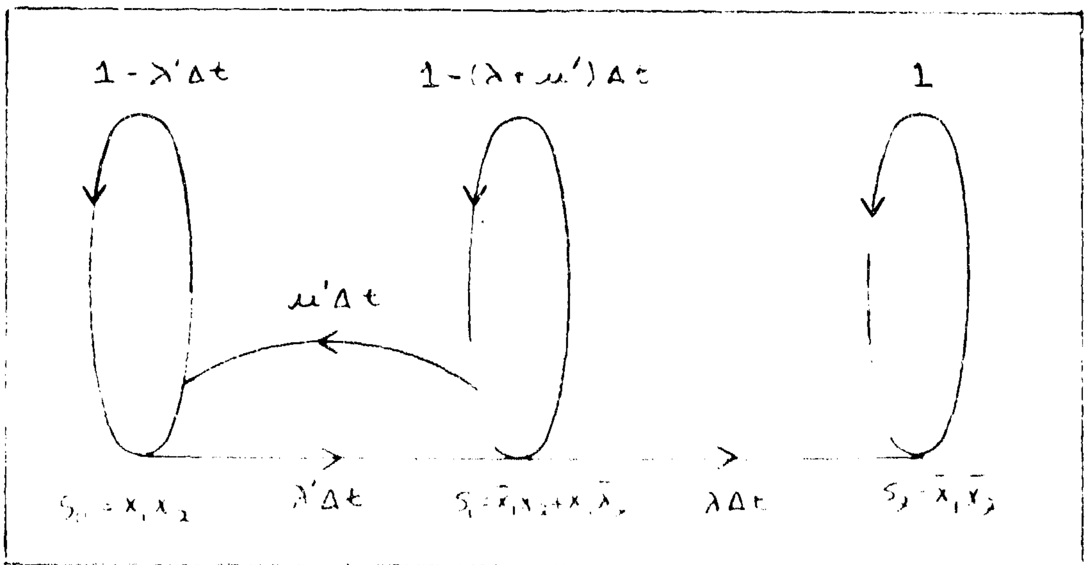

Figure 3.3 Markov Graph of Two-Unit Parallel System Reliability with Repair (Shooman, 1968:341)

The symbols in Figure 3.3 are defined as follows:

s_0 = both units operational

s_1 = one unit failed, one unit operational

s_2 = both units failed

λ = 1/Mean Time Between Failure or $1/\theta$

μ = 1/Mean Time To Repair or $1/\phi$

$\lambda' = 2\lambda$ for an ordinary system

$\lambda' = \lambda$ for a standby system

$\mu' = \mu$ for one repairman

$\mu' = k\mu$ for more than one repairman ($k > 1$)

Δt = change in time

Again, the Markov graph accounts for the failure of zero, one, or both units and the effects of differing number of repairmen working on the system. Note, however, that once both units are in a failed state simultaneously, state s_2 becomes an "absorbing" state; the system has reached the limit of it's reliability for that cycle. Once the entire system is repaired, the reliability cycle starts again.

The reliability equation used in this model is also derived from it's Markov graph (Shooman, 1968:341-342).

$$R(t) = Ps_0(t) + Ps_1(t) \tag{11}$$

where

$$Ps_0(t) = (\lambda + \mu + r_1/r_1 - r_2)e^{r_1 t} - (\lambda + \mu + r_2/r_1 - r_2)e^{r_2 t}$$

$$Ps_1(t) = (\lambda'/r_1 - r_2)e^{r_1 t} - (\lambda'/r_1 - r_2)e^{r_2 t}$$

$$r_1, r_2 = [-(\lambda + \lambda' + \mu') +/- ((\lambda + \lambda' + \mu')^2 - 4\lambda\lambda')**1/2]1/2$$

and other symbols are as defined in Figure 3.3

<u>Model</u>. The simulation model used the following baseline parameters:

Mean Time Between Failure ($1/\lambda$)	=	100 hours
Mean Time To Repair ($1/\mu$)	=	20 hours
Simulated Test Time	=	175 hours

Both the "true" sample MTBFs and MTTRs and the simulated sample MTBFs and MTTRs were drawn from the chi-square distribution in the same manner described for the single-unit system availability simulation. Also, when the same seeds for the random sample generation are used for both the availability and reliability simulation, they correspond to the same sample systems.

In summary, simulated availabilities and reliabilities were found for Type II censored sample sizes of 10, 20, 30, and 50. Point estimates were found for the lower confidence limits of 0.98, 0.95, 0.90, 0.85, and 0.80. The model ran for 500 repetitions with 500 trials per repetition.

IV. Monte Carlo Simulation Results

After the Monte Carlo simulation model was run as described in Chapter 3, the simulated point estimates were compared against the "true" lower confidence limit point estimates for the single-unit system availability and the two-unit parallel system availability and reliability. The success of using this Monte Carlo method is based upon the simulation coverages of the true lower confidence limit point estimates.

Single-Unit System Availability

The Monte Carlo simulation model was run to find estimated single-unit system steady-state availabilities using the following parameters:

Mean Time Between Failure = 100 hours

Mean Time To Repair = 20 hours

Thus, the exact availability is as follows:

$$A_{ss}(t) = 100/(100 + 20) = .8333$$

The Monte Carlo simulation coverages of this single-unit system steady-state availability are shown in Table I. Again, all simulations were run for 500 repetitions with 500 trials per repetition.

Table I.

Results of the Monte Carlo Method of Simulating

Single-Unit System Availabilities

Sample Size	Actual Coverage (%)				
	.98	.95	.90	.85	.80
10	97.6	93.8	90.2	86.2	80.0
20	97.4	95.2	90.0	86.0	81.6
30	96.6	93.4	88.6	83.4	76.8
50	98.6	95.6	91.2	86.6	81.4

Two-Unit Parallel System Availability

The Monte Carlo simulation of two-unit parallal system availabilities used these parameters:

Mean Time Between Failure = 100 hours

Mean Time To Repair = 20 hours

Testing Time = 175 hours

The exact availability (A(t) where t is 175 hours) for the two-unit parallel system is 0.9460. (The exact availability was found by solving Equation 10 in Chapter 3 using the above parameters.) The Monte Carlo simulation coverages for this two-unit parallel system availability are sh wn in Table II. All simulations were run for 500 repetitions with 500 trials per repetition.

Table II.

Results of the Monte Carlo Method of Simulating

Two-Unit Parallel System Availabilities

Sample Size	Actual Coverage (%)				
	.98	.95	.90	.85	.80
10	96.2	93.4	90.2	86.0	81.8
20	98.8	96.4	92.4	87.8	83.4
30	97.4	94.6	91.4	86.4	80.8
50	98.4	95.6	90.4	85.8	79.8

Two-Unit Parallel System Reliability

The Monte Carlo simulation of two-unit parallel system reliabilities used the following parameters:

Mean Time Between Failure = 100 hours

Mean Time To Repair = 20 hours

Testing Time = 175 hours

The exact reliability (R(t) where t is 175 hours) is 0.6565. (The exact reliability was found by solving Equation 11 in Chapter 3 using the above parameters.) Table III presents the Monte Carlo simulation coverages for the two-unit parallel system reliability.

Table III.

Results of the Monte Carlo Method of Simulating

Two-Unit Parallel System Reliabilities

Sample Size	Actual Coverage (%)				
	.98	.95	.90	.85	.80
10	99.6	96.6	92.4	89.2	84.4
20	99.4	97.8	94.8	90.4	87.0
30	98.8	96.2	92.4	88.6	83.8
50	98.8	94.8	89.6	85.4	80.8

Summary

In summary, Monte Carlo coverages were obtained for the lower confidence limits for single-unit system availability, the two-unit parallel system availability, and the two-unit parallel system reliability.

V. Conclusions and Recommendations

The primary objective of this research is to develop an efficient Monte Carlo simulation method to find lower confidence limits for the availability and reliability of maintained systems. After analyzing the results of the single-unit system availability simulation and the two-unit parallel system availability and reliability simulation, the conclusion is that this Monte Carlo simulation method is a viable and efficient method of simulating lower confidence limit system availability and reliability point estimates. However, it should be noted that the coverage of system reliability lower confidence limits is somewhat high, although still fairly good.

Recommendations for future research include to further study the Monte Carlo method of simulating system reliability lower confidence limits, to further study using this Monte Carlo method with varying parameters and different situations, and, finally, to design and develop an interactive, user-friendly program which allows varying inputs for different maintained systems.

In conclusion, the more research performed on accurately and efficiently simulating lower confidence limits for system availability and reliability, the more engineers can use this simulation tool to design and develop the most effective and reliable systems and equipment possible. Ultimately, the

better equipment and systems fielded to U. S. Armed Forces personnel, the better the country's defensive and war-fighting capabilities will be now and in the future.

Appendix A

Sampling Method

As mentioned in Chapter 3, "The Monte Carlo Simulation Model," sample θs, or Mean Time Between Failures (MTBFs), and sample ϕs, or Mean Time To Repairs (MTTRs), were drawn using the chi-square distribution as follows:

$$\theta = (X^2_{2r}\theta)/2r \qquad \text{(A1)}$$

where

X^2_{2r} = random number drawn from the chi-square distribution with 2r degrees of freedom

r = sample size

MTTRs were generated using the same equation with sub- stituted for θ.

Many methods of generating random numbers from the chi-square distribution are available, but only two methods were used in this research. The first method was to directly generate chi-square distributed random numbers using the International Mathematical Statistical Library (IMSL) GGCHS (chi-square random deviate generator) subroutine. The second method used was to generate a random number using the chi-square distribution's reproductive property (Miyamura, 1982:315). This method involved summing the absolute value of the natural logarithm of a uniform (0,1) random number r (r is the sample size) times. The uniform random number was

generated by the IMSL function GGUBFS which is a basic uniform (0,1) random number generator function.

There is virtually no difference between the two methods of generating chi-square distributed random numbers. Two simulations of sample size 10 were run, each using a different generation method and the same seed. The results are shown in Table IA and Table IIA.

Table IA.

Comparison of Coverages of the Two Random Number Generator Methods for the Single-Unit System Availability

Method	Actual Numbers and Coverage (%)				
	.98	.95	.90	.85	.80
GGCHS	97.6	93.8	90.2	86.2	80.0
	488	469	451	431	400
GGUBFS	97.6	93.8	90.2	86.2	80.0
	488	469	451	431	400

Table IIA.

Comparison of Coverages of the Two Random Number Generator Methods for the Two-Unit Parallel System Availability

Method	Actual Numbers and Coverage (%)				
	.98	.95	.90	.85	.80
GGCHS	96.2 481	93.4 467	90.2 451	86.0 431	81.6 408
GGUBFS	96.2 481	93.4 467	90.2 451	86.0 431	81.8 409

Appendix B

```
      PROGRAM AAAAA


      INTEGER A,B,C,D,N,E,R
      REAL    TTIME,MTBF,MTTR,CHI2
      REAL    LAMBDA,MU
      REAL    AROTS(500,5,3)
      REAL    ANTER(500,3)
      REAL    LAMBPR
      DOUBLE PRECISION DSEED

      DATA    AROTS/7500*0.0/
      DATA    ANTER/1500*0.0/

      OPEN(UNIT= ,FILE='     ')
      REWIND(UNIT= )

      DSEED = XXXXXXX
      TTIME = 175
      N = XX
      R = XX

      DO 100 A = 1,500
         CHI2 = 0.0
         DO 500 E = 1,R
            CHI2 = -(LOG(GGUBFS(DSEED))) + CHI2
500      CONTINUE

OR
         CALL GGCHS(DSEED,N,R,CHI2)

         MTBF = (CHI2*100)/N
         DO 600 E = 1,R
            CHI2 = -(LOG(GGUBFS(DSEED))) + CHI2
600      CONTINUE
         MTTR = (CHI2*20)/N
         DO 200 B = 1,500
            CALL GNEXP(DSEED,LAMBDA,MU,MTBF,MTTR)

            LAMBPR = LAMBDA * 2
            B2 = -(LAMBDA + LAMBPR + MU + MU)
            AC1 = ((-(B2))**2)
            AC2 = 4*(LAMBDA*LAMBPR + LAMPR*MU + MU*MU)
            AC  = SQRT(AC1 - AC2)
            R3 = (B2 + AC)/2
            R4 = (B2 - AC)/2
```

```
          PAR1 = (1-((LAMBDA*LAMBPR)/(R3*R4)))
          PAR2 = (LAMBDA*LAMBPR)/(R3-R4)
          PAR3 = ((EXP(R3*TTIME))/R3)-((EXP(R4*TTIME))/R4)
          AVAL = PAR1 - (PAR2*PAR3)

OR
          LAMBPR = LAMBDA * 2
          B2 = -(LAMBDA + LAMBPR + MU)
          AC = SQRT(((-(B2))**2)-(4*LAMBDA*LAMBPR))
          R1 = (B2 + AC)/2
          R2 = (B2 - AC)/2
          NUM1 = (LAMBDA+MU+R1)*(EXP(R1*TTIME))
          NUM2 = (LAMBDA+MU+R2)*(EXP(R2*TTIME))
          DEN = R1 - R2
          PS0 = (NUM1/DEN) - (NUM2/DEN)
          NUM3 = ((LAMBPR/DEN)*(EXP(R1*TTIME)))
          NUM4 = ((LAMBPR/DEN)*(EXP(R2*TTIME)))
          PS1 = NUM3 - NUM4
          REL = PS0 + PS1

OR
          R5 = MU/(LAMBDA + MU)
          R6 = LAMBDA/(LAMBDA + MU)
          AVALS = R5

          ANTER(B,1) = R1 OR R3 OR R5
          ANTER(B,2) = R2 OR R4 OR R6
          ANTER(B,3) = AVAL OR REL OR AVALS

          IF (B .EQ. 500) THEN
             CALL ORDER(ANTER)
             CALL RROOTS(ANTER,A,AROTS)
          ENDIF
200       CONTINUE
100    CONTINUE

       FORMAT(T2,F20.15,2X,F20.15,2X,F20.15)

       DO 300 C = 1,500
          DO 400 D = 1,5
             WRITE(UNIT= ,FMT=  )AROTS(C,D,1),AROTS(C,D,2),
                                 AROTS(C,D,3)
400       CONTINUE
300    CONTINUE

       REWIND(UNIT= )
       CLOSE(UNIT= )

       STOP
       END
```

```
      SUBROUTINE GNEXP(DSEED,LAMBDA,MU,MTBF,MTTR)

      INTEGER N,E,R
      REAL    LAMBDA,MU
      REAL    MTBF,MTTR
      REAL    CHI2
      REAL    THETA,PHI
      DOUBLE PRECISION DSEED

      N = XX
      R = XX

      CHI2 = 0.0
      DO 700 E = 1,R
         CHI2 = -(LOG(GGUBFS(DSEED))) + CHI2
700   CONTINUE
      THETA = (CHI2*MTBF)/N
      LAMBDA = 1/THETA

      CHI2 = 0.0
      DO 800 E = 1,R
         CHI2 = -(LOG(GGUBFS(DSEED))) + CHI2
800   CONTINUE
      PHI = (CHI2*MTTR)/N
      MU = 1/PHI

      RETURN
      END
```

```
      SUBROUTINE ORDER(INTERM)

      INTEGER C,SWITCH
      REAL    INTERM(500,3)
      REAL    TEMP1,TEMP2,TEMP3

      SWITCH = 1

10    IF (SWITCH .EQ. 1) THEN
         SWITCH = 0
         DO 100 C = 2,500
            IF (INTERM(C,3) .GT. INTERM(C-1,3)) THEN
               TEMP1 = INTERM(C,1)
               TEMP2 = INTERM(C,2)
               TEMP3 = INTERM(C,3)
               INTERM(C,1) = INTERM(C-1,1)
               INTERM(C,2) = INTERM(C-1,2)
               INTERM(C,3) = INTERM(C-1,3)
               INTERM(C-1,1) = TEMP1
               INTERM(C-1,2) = TEMP2
               INTERM(C-1,2) = TEMP3
               SWITCH = 1
            ENDIF
100      CONTINUE
         GOTO 10
       ENDIF

      RETURN
      END
```

```
      SUBROUTINE RROOTS(RINTER,A,ROTS)

      INTEGER A,C,D
      REAL    RINTER(500,3),ROTS(500,5,3)
      REAL    LCB,PROPOR,ROOT1,ROOT2,I
      REAL    POLATE,DIFF

      DO 100 D = 1,5
         IF (D .EQ. 1) THEN
            I = 490.692
         ENDIF
         IF (D .EQ. 2) THEN
              = 475.68
         ENDIF
         IF (D .EQ. 3) THEN
            I = 450.66
         ENDIF
         IF (D .EQ. 4) THEN
            I = 425.64
         ENDIF
         IF (D .EQ. 5) THEN
            I = 400.62
         ENDIF

         DO 200 C = 1,500

            LCB = RINTER(500,3)
            ROOT1 = RINTER(500,1)
            ROOT2 = RINTER(500,2)

            IF (I .LT. C) THEN
               PROPOR = I - (C-1)
               POLATE = RINTER(C,3) - RINTER(C-1,3)
               DIFF = PROPOR * POLATE
               LCB = DIFF + RINTER(C-1,3)
               ROOT1 = RINTER(C-1,1)
               ROOT2 = RINTER(C-1,2)
               GOTO 300
            ENDIF

200      CONTINUE

300      ROTS(A,D,1) = ROOT1
         ROTS(A,D,2) = ROOT2
         ROTS(A,D,3) = LCB

100   CONTINUE

      RETURN
      END
```

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This research determined the feasibility and efficiency of a Monte Carlo method of simulating the lower confidence limits for the availabilities and reliabilities of maintained systems. The steady-state availabilities of single-unit systems and the time-constrained availabilities and reliabilities of two-unit parallel systems were simulated.

First, a baseline of "true" exponentially-distributed Mean Time Between Failures (MTBFs) and Mean Time To Repairs (MTTRs) were simulated using the chi-square distribution. Then other MTBFs and MTTRs were simulated to represent sampling of other systems. The availabilities and reliabilities were found using these simulated MTBFs and MTTRs. Next, simulated availabilities and reliabilities were ordered, and lower confidence limits were found. These lower confidence limit point estimates were compared against the systems' exact availabilities and reliabilities. Lastly, the success of this Monte Carlo method is determined by how well the simulated lower confidence limit availability and reliability point estimates cover the exact availabilities and reliabilities.